SAT-based Reasoning Techniques for LTL over Finite and Infinite Traces

Jianwen Li

East China Normal University
Main work done at Rice University

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Linear Temporal Logic

- First introduced to Computer Science by A. Pnueli in 1977
- Formal verification (over infinite traces: LTL)
- AI (over finite traces : LTL_f)

Linear Temporal Logic

Syntax for LTL and LTL_f

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid X\phi \mid \phi U \phi \mid \phi R \phi \mid G\phi \mid F\phi;$$

- $\neg(\phi_1 U \phi_2) = \neg\phi_1 R \neg\phi_2$
- $\neg X\phi = N\neg\phi$ (Weak Next), for LTL_f only
- $F\phi = tt U \phi$
- $G\phi = ff R \phi$

Linear Temporal Logic

Semantics for LTL (LTL_f),

- Let ξ be a trace with $|\xi| = n$ ($n > 0$)
  - $\xi \models p$ if $p \in \xi[0]$
  - $\xi \models \neg\phi$ if $\xi \not\models \phi$
  - $\xi \models \phi_1 \wedge \phi_2$ if $\xi \models \phi_1$ and $\xi \models \phi_2$
  - $\xi \models X\phi$ if $n > 1$ and $\xi_1 \models \phi$
  - $\xi \models \phi_1 U \phi_2$ if (1) there is $0 \leq i < n$ s.t. $\xi_i \models \phi_2$ and (2) for every $0 \leq j < i$ it holds $\xi_j \models \phi_1$.
- LTL semantics: $n = \propto$
- LTL_f semantics: $n < \propto$

LTL vs. LTL_f

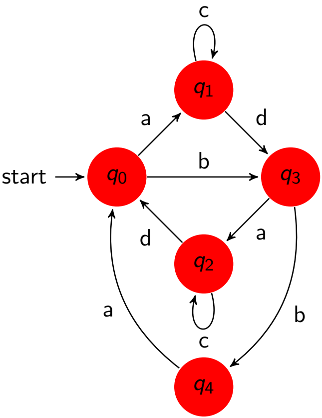
- $X tt$ is always true in LTL, but not in LTL_f
- $(a \wedge X tt) \not\equiv a$ in LTL_f
- $\neg X\phi \neq X\neg\phi$ $(\neg X\phi = N\neg\phi)$
- $GX\phi$ is never satisfiable in LTL_f

This Talk

- Present an on-the-fly approach to construct automata by SAT solvers
- Show one of its applications to solve LTL_f satisfiability checking problem

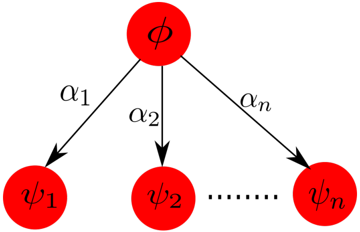
LTL($_f$) to Automata

$\phi \implies$



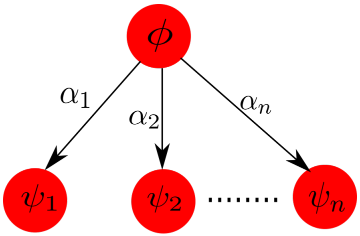
LTL($_f$) to Automata

$\phi \iff \bigvee_i (\alpha_i \wedge X(\psi_i))$



LTL($_f$) to Automata

$$\phi \Longleftrightarrow \bigvee_i (\alpha_i \wedge X(\psi_i))$$



Bottleneck: $\phi \Longleftrightarrow \bigvee_i (\alpha_i \wedge X(\psi_i))$ is expensive!

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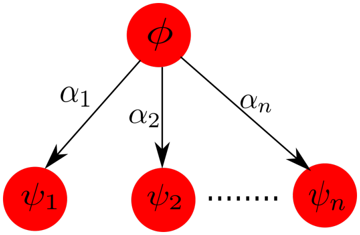

Bottleneck: $\phi \iff \bigvee_i(\alpha_i \wedge X(\psi_i))$ is expensive!
Question: Is it possible to generate ONE successor at one time (on the fly)?

neXt Normal Form (XNF)

If considering the temporal subformulas as atoms, an LTL($_f$) formula ϕ becomes a Boolean formula.

We say ϕ is in XNF iff only X/N subformulas can be its atom.

neXt Normal Form (XNF)

Example

- $(a \vee bUc) \wedge cRd$ is not in XNF
- $(a \vee c \vee b \wedge X(bUc) \wedge d \wedge (c \vee X(cRd))$ is in XNF

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3. Take $xnf(\phi)$ as the input of a SAT solver
4. SAT solver may return: $\{a, \neg b, X(aUb), X(\neg bU\neg a)\}$
   - A transition in the automaton is $\phi \xrightarrow{a\wedge\neg b} (aUb) \wedge (\neg bU\neg a)$

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To construct the whole automaton, just do enumeration!

LTL_f Satisfiability Checking

Problem

Given an LTL_f formula ϕ, is there a finite trace ξ s.t. $\xi \models \phi$?

- Fa is satisfiable;
- $Fa \wedge G\neg a$ is unsatisfiable;

Determining Final State

Introduce a new atom *Tail* to associate with X

$$X\psi \Longrightarrow \neg Tail \wedge X\psi$$

Example

- $b \vee X(aUb) \Longrightarrow b \vee \neg Tail \wedge X(aUb)$
- $XX\phi \Longrightarrow \neg Tail \wedge X(\neg Tail \wedge X\phi)$

Determining Final State

Theorem

ϕ is a final state iff $\mathit{Tail} \wedge \mathit{xnf}(\phi)$ is satisfiable (Boolean formula).

BLSC: Basic SAT-based Checking

Algorithm 1 BLSC: Basic on-the-fly LTL_f Satisfiability Checking

Require: An LTL_f formula ϕ.

Ensure: SAT or UNSAT.

if $Tail \wedge xnf(\phi)$ is satisfiable **then**
    return SAT;
end if
Add ϕ into *block_list*;
Let $\psi = xnf(\phi) \wedge \neg X(block\_list)$;
while ψ is satisfiable **do**
    Let t be a propositional assignment for ψ;
    Let $\phi' = \bigwedge\{\theta | X\theta \in t\}$;
    if BLSC (ϕ') returns SAT **then**
        return SAT;
    end if
end while
return UNSAT;

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  - BLSC leverages the satisfiability from SAT solvers
  - BLSC discards the unsatisfiability from SAT solvers

CDLSC: Conflict-Driven Checking

$\mathit{Tail} \wedge \mathit{xnf}(\phi)$ is UNSAT

$$\Longrightarrow$$

there is ψ s.t $\psi \subseteq \phi$[1] and $\mathit{Tail} \wedge \mathit{xnf}(\psi)$ is still unsatisfiable (UNSAT core)

[1] Consider ϕ have the form of $\bigwedge \phi_i$

CDLSC: Conflict-Driven Checking

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$$\Longrightarrow$$

there is ψ s.t $\psi \subseteq \phi$[1] and $Tail \wedge xnf(\psi)$ is still unsatisfiable (UNSAT core)

- ψ can be provided by a SAT solver
- ψ represents a set of states that cannot reach a final state in 0 step

[1] Consider ϕ have the form of $\bigwedge \phi_i$

CDLSC: Conflict-Driven Checking

Conflict Sequence: $\mathcal{C} = C_0, C_1, C_2, \ldots, C_k (k \geq 0)$

- C_i : a set of states that cannot reach a final state in i steps

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- Guided search:
  - ψ is known in some $C_i \implies$ a next state of ψ not in C_i is preferred: $xnf(\phi) \wedge \neg XC_i$
  - Otherwise, check final state: $Tail \wedge xnf(\psi)$

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- Unsatisfiability check: $\bigcap_{0 \leq j \leq i} C_j \subseteq C_{i+1}$

Evaluation

- Benchmarks: 7446 LTL-as-LTL_f formulas in previous work
- Platform: Rice Davinci cluster
- Timeout: 60 seconds for each instance
- Tools:
  1. Aalta-finite [LZPVH14]
  2. Aalta-infinite [LZPV15]
  3. ltl2sat [FG16]
  4. nuXmv (IC3+KLIVE) [CCDGMMMRT14]
  5. BLSC
  6. CDLSC

Evaluation

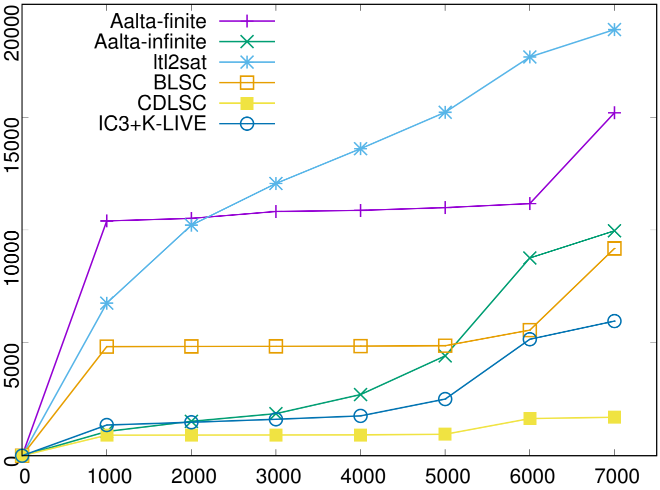

Figure 1: Cactus plot for LTL_f Satisfiability Checking on LTL-as-LTL_f Benchmarks.

Beyond Satisfiability Checking

- On-the-fly Synthesis for LTL over finite traces [AAAI2019]
- Satisfiability checking for LTL over infinite traces[FMSD 2019]
- Synthesis for LTL over infinite traces?
- On-the-fly LTL model checking?
- Extension to word-level?

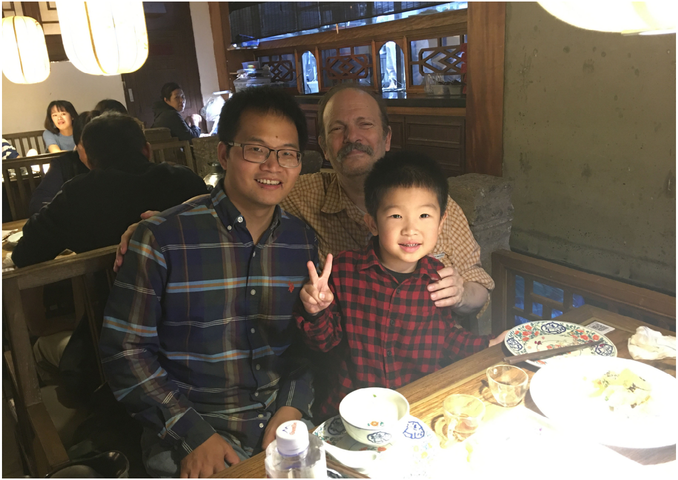

Figure 2: Shanghai, Oct. 2019.

Hope to see you again VERY soon!